resonance Documentation

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Contents:

Resonance is a companion software library to an interactive textbook written for an upper level undergraduate introduction to mechanical vibrations course.

The library is specifically designed for learning engineering principles through computational thinking and computational experimentation and thus we have guidelines on its design to facilitate this. Those guidelines are roughly:

- Don't teach programming for the sake of teaching programming. Show students how to solve problems and introduce programming along the way to solve those problems.
- Hide the fine details of programming and only use simple constructs, so that learning vibrations is highlighted instead of programming.
- Hide the simulation details (linear/nonlinear ODE solutions), but allow them to be exposed if needed.
- The software design is centered around the "System" object. Systems represent real things: a car, a bridge, a bicycle, an airplane wing.
- Students can use and construct systems.
- Students only create functions, no need to understand classes and their construction.
- Easy visualizations (time history plots and animations of systems)
- Extra informative and lots of error messages (try to predict student mistakes)

CHAPTER 1

Topical Outline

The course is taught over 20 two hour class periods during a quarter system of 10 weeks of instructions and 1 week for examinations. One of the 20 class periods is reserved for a midterm examination, 2 hours are reserved for exam reviews leaving 36 hours of in class time. The following lists the topics for each of the class periods which correspond to the detailed headers below:

L#	Date	Notebook #
01	W Sep 27	1, 2
02	M Oct 02	3
03	W Oct 04	4
04	M Oct 09	5
05	W Oct 11	6
06	M Oct 16	7
07	W Oct 18	8
08	M Oct 23	9
NA	T Oct 24	Drop Date
09	W Oct 25	10
10	M Oct 30	11
11	W Nov 01	12
12	M Nov 06	Exam
13	W Nov 08	13
NA	F Nov 10	Veterans Day Holiday
14	M Nov 13	14
15	W Nov 15	15
16	M Nov 20	16
17	W Nov 22	17
NA	R Nov 23	Thanksgiving Holiday
NA	F Nov 24	Thanksgiving Holiday
18	M Nov 27	18
19	W Nov 29	19
20	M Dec 04	20
21	W Dec 06	21
NA	T Dec 12	Final Exam @ 6:00 PM

1.1 Analyzing Vibrating Systems

1.1.1 1. Introduction to Jupyter

This notebook introduces students to the Jupyter notebook environment and establishes good practices for creating computational notebooks and scientific python programming.

After the completion of this assignment students will be able to:

- open Jupyter notebooks and operate basic functionality
- fetch assignments, complete exercises, submit work and view the graded work
- solve basic scientific python problems
- create a well formatted and fully executing notebook

1.1.2 2. Introduction to vibrations: Book Balancing on a Cup

This notebook introduces a single degree of freedom vibratory system in which a textbook balances on a cylindrical cup. The system is implemented as a model that students can interact with in order to visualize its free response and compare to the demonstration in the classroom.

After the completion of this assignment students will be able to:

- visualize a system's free response
- estimate the period of a sinusoidal vibration from a time series
- compare a computer simulation result to experimental result
- interactively adjust the book inertia to see the affect on system response
- understand the concept of natural frequency nd its relationship to mass/inertia

1.1.3 3. Measuring a Bicycle Wheel's Inertia

This notebook introduces the concept of using vibratory characteristics to estimate parameters of an existing system. It discusses how vibrations can be measured and how these measurements might relate to parameters of interest, such as the inertia of a bicycle wheel.

After the completion of this assignment students will be able to:

- describe different methods of measuring vibrations
- choose appropriate sensors and sensor placement
- visualize the vibrational measurements
- use curve fitting to estimate the period of oscillation
- understand the concept of natural frequency and its relationship to mass/inertia and stiffness
- state two of the three fundamental characteristics that govern vibration (mass/inertia and stiffness)
- use frequency domain techniques to characterize a system's behavior

1.1.4 4. Clock Pendulum with Air Drag Damping

This notebook introduces the third fundamental characteristic of vibration: energy dissipation through damping. A simple pendulum model is implemented that allows students to vary the damping parameters and visualize the three regimes of linear damping.

After the completion of this assignment students will be able to:

- understand the concept of damped natural frequency and its relationship to mass/inertia, stiffness, and damping
- state the three fundamental characteristics that make a system vibrate
- compute the free response of a linear system with viscous-damping in all three damping regimes
- identify critically damped, underdamped, and overdamped behavior
- determine whether a linear system is over/under/critically damped given its dynamic properties
- understand the difference between underdamping, overdamping, and crticial damping

1.1.5 5. Clock Pendulum with Air Drag and Joint Friction

This notebook builds on the previous one by introducing nonlinear damping through Coulomb friction. Students will be able to work with both a linear and nonlinear version of the same system (pendulum) in order to compare the free response in both cases.

After the completion of this assignment students will be able to:

- identify the function that governs the decay envelope

- compare this non-linear behavior to the linear behavior
- estimate the period of oscillation
- compute the free response of a non-linear system with viscous and coulomb damping

1.1.6 6. Vertical Vibration of a Bus Driver's Seat

This notebook introduces external forcing of a vibratory system, where the external force is modeled as a sinusoidal input to the bottom of a bus driver's seat.

After the completion of this assignment students will be able to:

- excite a system with a sinusoidal input
- understand the difference in transient and steady state solutions
- use autocorrelation to determine period
- relate the frequency response to the time series
- create a frequency response plot
- define resonance and determine the parameters that cause resonance

1.1.7 7. Vertical vibration of a Bus Driver's Seat with a Leaf Spring

This notebook builds on the previous one by replacing the linear spring with a realistic leaf spring.

After the completion of this assignment students will be able to:

- create a force versus displacement curve for a leaf spring
- describe the time response and frequency response of a non-linear system
- show that sinusoidal fitting does not necessarily describe non-linear vibration

1.1.8 8. Bicycle Lateral Vibration

This notebook introduces a simple lean and steer bicycle model as an example of a system with multiple degrees of freedom. Coupling and modes are discussed from a data analysis perspective.

After the completion of this assignment students will be able to:

- get a sense of the coupling of input to output through frequency response plots
- simulate a 2 DoF vibratory model
- identify a MDoF system and see effects of coupling through time and frequency domain
- determine if a general 2 DoF is stable
- sweep through input frequencies to discover modal frequencies

1.1.9 9. Simulating a building during an earthquake

This notebook uses a lumped parameter multi-story building model as a many-degree-of-freedom system with all oscillatory modes.

After the completion of this assignment students will be able to:

- examine time domain and frequency coupling with MDoF
- sweeping through frequencies to discover modal frequencies
- visualize the system's response at modal frequencies to see mode shapes

1.2 Modeling Vibrating Systems

1.2.1 10. Modeling the Bicycle Wheel Inertia Measurement System

This notebook walks through modeling two different test rigs for determining the vibrational characteristics of a bicycle wheel. After coming up with a simple model the students will use the canonical linear form of the equations of motion to derive various vibrational parameters.

After the completion of this assignment students will be able to:

- derive the equations of motion of a compound pendulum with Lagrange's method
- derive the equations of motion of a torsional pendulum with Lagrange's method
- linearize the compound pendulum equation
- put equations in canonical form
- review solutions to ODEs

1.2.2 11. Modeling a non-linear spring

TODO : Think this out more.

After the completion of this assignment students will be able to:

- will be able to derive the nonlinear euqations of motion of a system with simple kinmeatics with lagrange's method

1.2.3 12. Modeling the car on the bumpy road

Here will will present the base excitation single degree of freedom system and the students will derive the equations of motion. They will then explore the displacement and force transmisiblity frequency response functions.

After the completion of this assignment students will be able to:

- derive the linear equations of motion ofa system with simple kinematics using lagrange's method
- create system object with custom euqations of motion an simulate the system

1.2.4 13. Modeling the book on a cup

The book balancing on the cup will be revisited. The students will derive the equations of motion which require more complex kinematic analysis and explore the analytical equations of motion. The stability thresholds will be determined as well as the period from the linear model.

After the completion of this assignment students will be able to:

- derive the euqations of motion of a system with non-trivial kinematics with lagrange's method
- apply a linearization procedure to non-linear equations of motion

- determine the stability of a linear system analytically and verify through simulation

1.2.5 14. Balancing your car tire at the autoshop

The mass imbalance problem will be presented through the analytical model of an unbalance car tire. The frequency response will be derived and examined.

After the completion of this assignment students will be able to:

- derive the equations of motion fo a mass imbalance system

1.2.6 15. Engine cam non-sinusoidal periodic forcing

Using an engine cam piecewise periodic function the students will learn how a Fourier series can be used to find the solution to the differential equations symbolicaly.

After the completion of this assignment students will be able to:

- generate a Fourier series of a periodic function
- find the analytic solution of the the mass-spring-damper system

1.2.7 16. Modeling a bulding during an earthquake

We will revisit the multi-story building model and derive the equations of motion for the system. The students will use eigenanalysis of the simple system to discover the modes of motion and simulate the behavior.

After the completion of this assignment students will be able to:

- perform modal analysis of the system to determine its modal frequencies and mode shapes
- represent model using a matric equation of motion (canoncial form)
- formulate the equations of motion for a MDoF system
- use eignvalue analyssis to determine the modeshapes of a mDoF system
- plot the motion of a MDoF system (with no damping) using the analytical solution
- form a MDoF model corresponding to a chain of floors in a buliding

1.2.8 17. Bicycle Model

The students will be given the analytical canocial form of the bicycle equations that do not have simple damping. They will have to convert to state space form and do a full eigenanalysis of the general form. The modes will be examined and the nature of the bicycle motion discovered.

After the completion of this assignment students will be able to:

- convert the canonical linear form into state space form
- interpret eigenvalues and eienvectors of a general 2 DoF linear system

1.3 Designing Vibrating Systems

1.3.1 18. Design a Clock that Keeps Time

The students will be presented with a compound pendulum model of a clock's bob that does not keep time well due to friction and air drag. They will be tasked with designing a system that adds in the right amount of additional energy so that the pendulum has the desired constant period.

After the completion of this assignment students will be able to:

- develop an analytic model of a energy injection system
- simulate the motion of clock and determine its time varying period
- choose the energy injection system parameters that will cause the clock to work as intended

1.3.2 19. Isolator Selection

The students will be presented with a model of X and asked to select and/or design a commercially available vibration isolator that ensures the system meets specific vibrational design criteria.

After the completion of this assignment students will be able to:

- discuss and justify trade-offs and design decisions
- model the system with additional damping provided by isolation
- select/design a vibration isolator to meet given vibration specifications
- analyze a system's motion to determine its vibrational characteristics

1.3.3 20. Designing a Tuned Mass Damper to Earthquake Proof a Building

Students will be presented with a single (or multi?) floor building model. They will need to modify the model to includes a laterally actuated mass on the roof. They will be asked to design an actuation scheme that prevents the building from having too large of displacements or resonance while excited by a earthquake-like vibration at its base.

After the completion of this assignment students will be able to:

- add a generic vibration absorber to a building model
- use a building model to simulate the motion of a building without damping
- choose design criteria for the building and justify decisions (with ISO standards)
- design an absorber that meets their design criteria
- use the frequency response function to demonstrate the effect of the vibration absorber

1.3.4 21. Designing a stable bicycle

The students will be presented with a 2 DoF linear model of a bicycle in canonical form with analytical expressions for the M, C, and K matrix entries that are functions of the 25 bicycle parameters. The students will be asked to discover bicycle designs that meet certain criteria through eigenanalysis and simulation.

After the completion of this assignment students will be able to:

- determine parameters which cause the 2 DoF system to be stable/unstable

- simulate and visualize the motion of a bicycle with difference parameters
- determine and describe the influence of the physical parameters, initial conditions, and steering input on the dynamics of the vehicle
- design a bicycle that meets specific design criteria

1.3.5 22. Designing Shock Absorbtion for a Car

The students will be presented with 2D planar data generated from a "ground truth" 3 DoF half car model. Their job will be to design a quarter car model that behaves similarly to the ground truth model. Once they have a working simple model, then they will design an improved shock absorber for the quarter car model using analytic and computational methods. The instructors will then provide the students with the ground truth model, i.e. the "real" car, and the students will need to show that the ride quality is improved and that design criteria is met.

After the completion of this assignment students will be able to:

- develop a simple analytic model that predicts motion provided from planar 2D "experimental" data
- select springs and dampers to meet given design criteria by demonstrating performance with the simple analytic model
- demonstrate that the designed shock absorber works well for the "real" car
- discuss why the design does or does not meet the design criteria
- reflect on their modeling and design decisions after having tested it against the ground truth model

CHAPTER 2

Creating and Exercising a Custom Single Degree of Freedom System

Note: You can download this example as a Python script: custom-sdof-system.py or Jupyter notebook: custom-sdof-system.ipynb.

2.1 Creating a new system

The first step is to import a "blank" `SingleDoFLinearSystem` and initialize it.

```
from resonance.linear_systems import SingleDoFLinearSystem

msd_sys = SingleDoFLinearSystem()
```

Now define the constant variables for the system. In this case, the single degree of freedom system will be described by its mass, natural frequency, and damping ratio.

```
msd_sys.constants['m'] = 1.0  # kg
msd_sys.constants['fn'] = 1.0  # Hz
msd_sys.constants['zeta'] = 0.1  # unitless

msd_sys.constants
```

```
{'m': 1.0, 'fn': 1.0, 'zeta': 0.1}
```

Define the coordinate and speed. The software assumes that the speed is defined as the time derivative of the coordinate, i.e. $v = \dot{x}$.

```
msd_sys.coordinates['x'] = 1.0  # m
msd_sys.speeds['v'] = 0.0  # m/s
```

```
msd_sys.coordinates
```

```
{'x': 1.0}
```

```
msd_sys.speeds
```

```
{'v': 0.0}
```

```
msd_sys.states
```

```
{'x': 1.0, 'v': 0.0}
```

Now that the coordinate, speed, and constants are defined the equations of motion can be defined. For a single degree of freedom system a Python function must be defined that uses the system's constants to compute the coefficients to the canonical second order equation, $m\dot{v} + cv + kx = 0$.

The inputs to the function are the constants. The variable names should match those defined above on the system.

```
import numpy as np

def calculate_canonical_coefficients(m, fn, zeta):
    """Returns the system's mass, damping, and stiffness coefficients given
    the system's constants."""
    wn = 2*np.pi*fn
    k = m*wn**2
    c = zeta*2*wn*m
    return m, c, k

msd_sys.canonical_coeffs_func = calculate_canonical_coefficients
```

Once this function is defined and added to the system m, c, k can be computed using:

```
msd_sys.canonical_coefficients()
```

```
(1.0, 1.2566370614359172, 39.47841760435743)
```

The period of the natural frequency can be computed with:

```
msd_sys.period()
```

```
1.005037815259212
```

All information about the system can be displayed:

```
msd_sys
```

```
System name: SingleDoFLinearSystem

Canonical coefficients function defined: True
Configuration plot function defined: False
Configuration update function defined: False

Constants
=========
```

(continues on next page)

(continued from previous page)

```
m = 1.00000
fn = 1.00000
zeta = 0.10000

Coordinates
===========
x = 1.00000

Speeds
======
v = d(x)/dt = 0.00000

Measurements
============
```

2.2 Simulating the free response

The `free_response()` function simulates the now fully defined system given as an initial value problem. One or both of the coordinates and speeds must be set to provide a free response. The following shows the response to both x and v being set to some initial values.

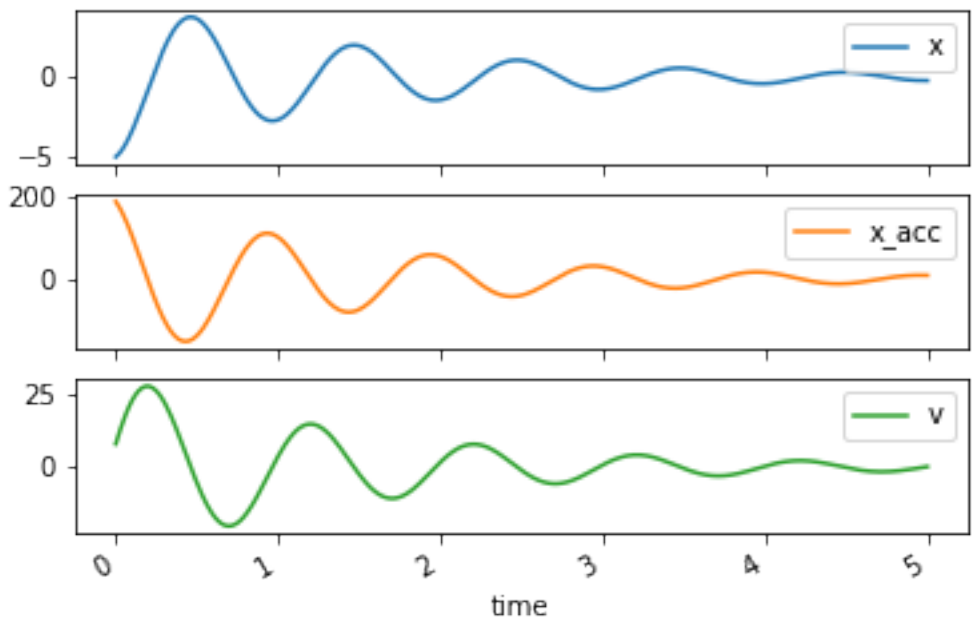
```
msd_sys.coordinates['x'] = -5.0
msd_sys.speeds['v'] = 8.0
```

`free_response()` returns a Pandas `DataFrame` with the time values as the index and columns for the coordinate, speed, and additionally the time derivative of the speed (acceleration in this case). See https://pandas.pydata.org/pandas-docs/stable/getting_started/dsintro.html for an introduction to `DataFrame`.

```
trajectories = msd_sys.free_response(5.0)
trajectories
```

There are a variety of plotting methods associated with the `DataFrame` that can be used to quickly plot the trajectories of the coordinate, speed, and acceleration. See more about plotting `DataFrames` at https://pandas.pydata.org/pandas-docs/stable/user_guide/visualization.html.

```
axes = trajectories.plot(subplots=True)
```



2.2.1 Response to change in constants

This system is *parameterized* by its mass, natural frequency, and damping ratio. It can be useful to plot the trajectories of position for different values of ζ for example.

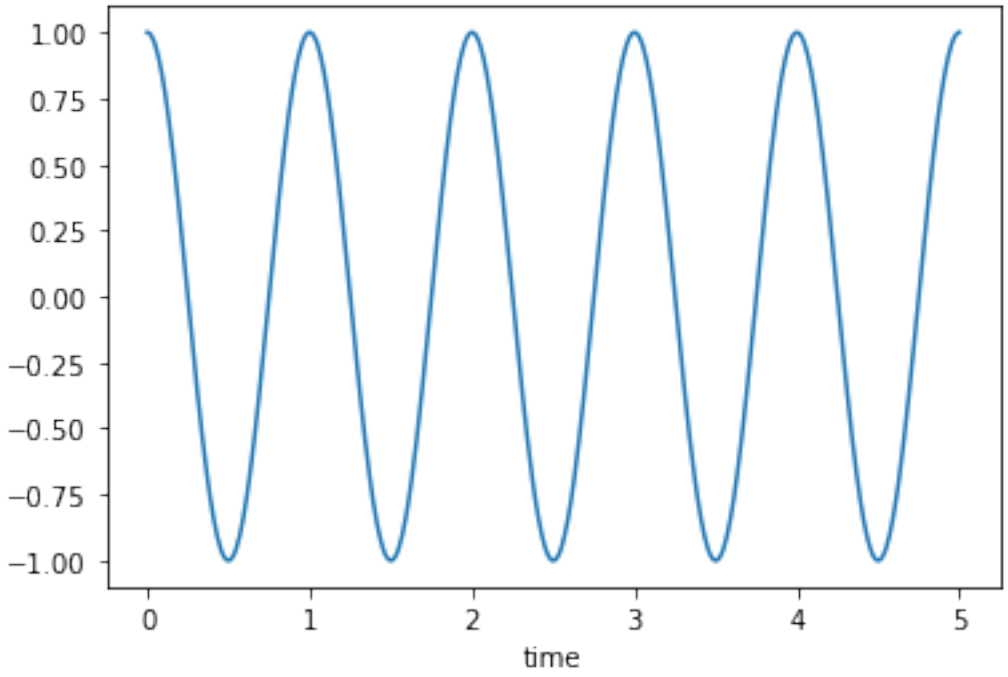
Set the initial conditions back to simply stretching the spring 1 meter:

```
msd_sys.coordinates['x'] = 1.0
msd_sys.speeds['v'] = 0.0
```

Now change ζ to different values and simulate the free response to see the different damping regimes:

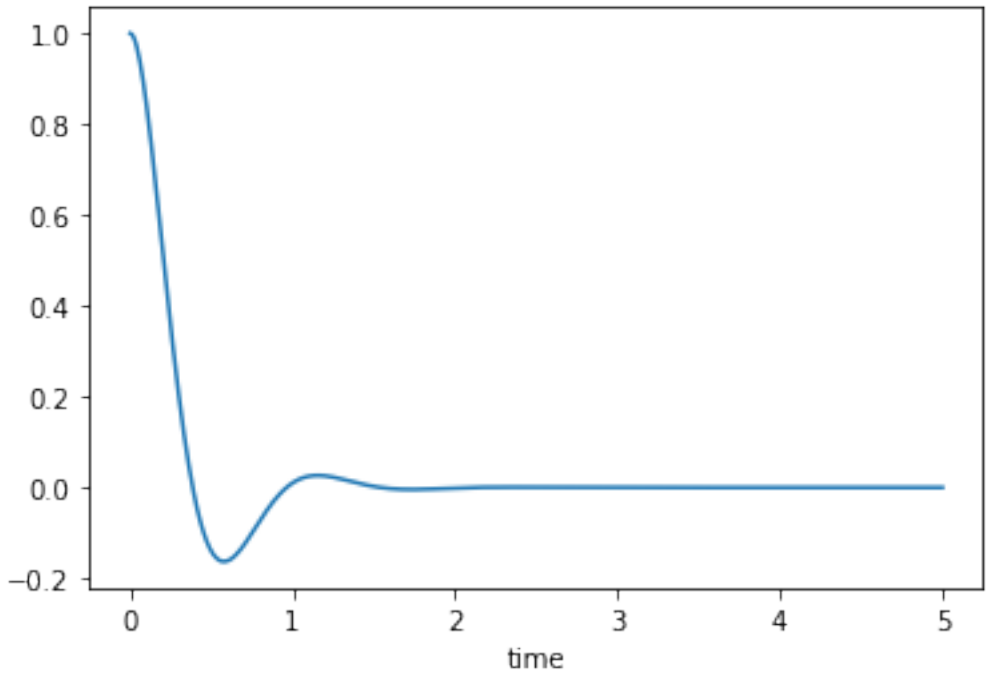
Un-damped, $\zeta = 0$

```
msd_sys.constants['zeta'] = 0.0  # Unitless
trajectories = msd_sys.free_response(5.0)
axes = trajectories['x'].plot()
```



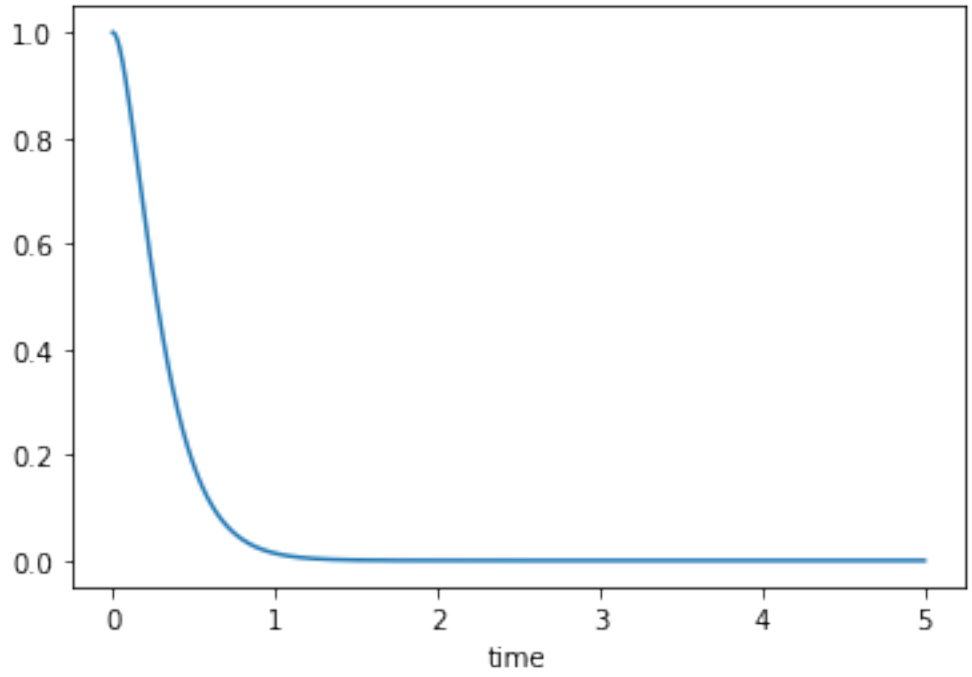
Under-damped, $0 < \zeta < 1$

```
msd_sys.constants['zeta'] = 0.5  # Unitless
trajectories = msd_sys.free_response(5.0)
axes = trajectories['x'].plot()
```



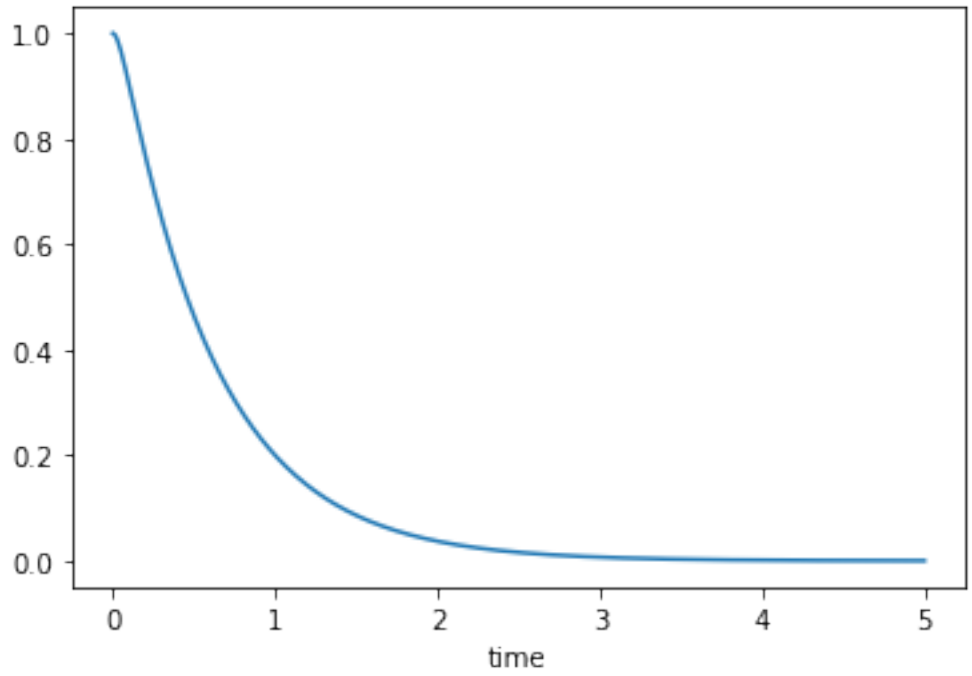
Critically damped, $\zeta = 1$

```
msd_sys.constants['zeta'] = 1.0  # Unitless
trajectories = msd_sys.free_response(5.0)
axes = trajectories['x'].plot()
```



Over-damped, $\zeta > 1$

```
msd_sys.constants['zeta'] = 2.0  # Unitless
trajectories = msd_sys.free_response(5.0)
axes = trajectories['x'].plot()
```



2.3 Adding measurements

It is often useful to calculate the trajectories of other quantities. Systems in resonance allow "measurements" to be defined. These measurements are functions of the constants, coordinates, speeds, and/or time. To create a new measurement, create a function that returns the quantity of interest. Here a measurement function is defined that calculates the kinetic energy ($\frac{1}{2}mv^2$) of the system and then added to the system with variable name `KE`.

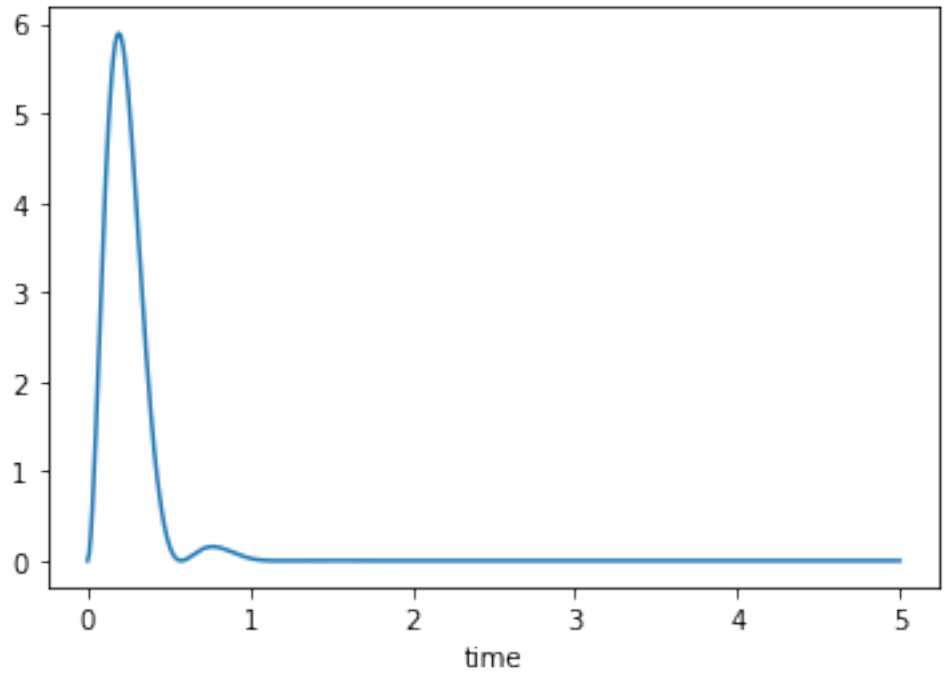
```
def calculate_kinetic_energy(m, v):
    return m*v**2/2

msd_sys.add_measurement('KE', calculate_kinetic_energy)
```

Once added, the measurement will be computed and added to the `DataFrame` containing the trajectories:

```
msd_sys.constants['zeta'] = 0.5  # Unitless
trajectories = msd_sys.free_response(5.0)
trajectories
```

and can be plotted like any other column:

```
axes = trajectories['KE'].plot()
```



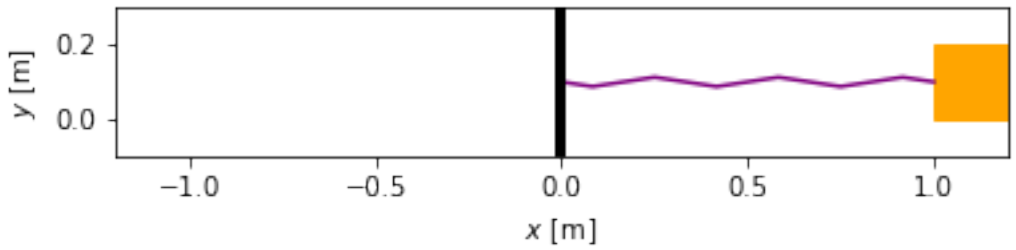
2.4 Plotting the configuration

`resonance` systems can plot and animate at the system's configuration. To do so, a custom function that generates a configuration plot using matplotlib must be defined and associated with the system. Below a plot is created to show an orange block representing the mass and a spring attached to the block. The `spring()` function conveniently provides the x and y data needed to plot the spring.

```
import matplotlib.pyplot as plt
from resonance.functions import spring

# create a new constant to describe the block's dimension, l
msd_sys.constants['l'] = 0.2  # m

def create_configuration_figure(x, l):

    # create a figure with one or more axes
    fig, ax = plt.subplots()

    # the `spring()` function creates the x and y data for plotting a simple
    # spring
    spring_x_data, spring_y_data = spring(0.0, x, l/2, l/2, l/8, n=3)
    lines = ax.plot(spring_x_data, spring_y_data, color='purple')
    spring_line = lines[0]

    # add a square that represents the mass
    square = plt.Rectangle((x, 0.0), width=l, height=l, color='orange')
    ax.add_patch(square)

    # add a vertical line representing the spring's attachment point
    ax.axvline(0.0, linewidth=4.0, color='black')

    # set axis limits and aspect ratio such that the entire motion will appear
    ax.set_ylim((-l/2, 3*l/2))
    ax.set_xlim((-np.abs(x) - l, np.abs(x) + l))
    ax.set_aspect('equal')

    ax.set_xlabel('$x$ [m]')
    ax.set_ylabel('$y$ [m]')

    # this function must return the figure as the first item
    # but you also may return any number of objects that you'd like to have
    # access to modify, e.g. for an animation update

    return fig, ax, spring_line, square

# associate the function with the system
msd_sys.config_plot_func = create_configuration_figure
```

Now the configuration plot can be generated with `plot_configuration()`. This returns the same results as the function defined above.

```
fig, ax, spring_line, square = msd_sys.plot_configuration()
```



2.5 Animating the configuration

Reset to un-damped motion and simulate again

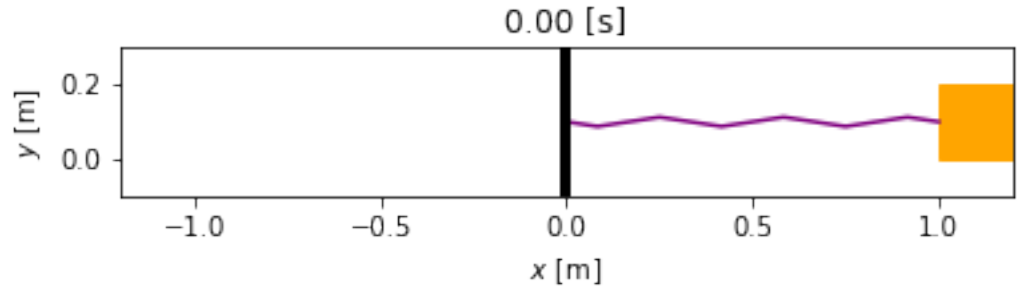
```
msd_sys.constants['zeta'] = 0.1
trajectories = msd_sys.free_response(5.0)
```

To animate the configuration, create a function that updates the various matplotlib objects using any constants, coordinates, speeds, and/or the special variable `time`. The last input arguments to this function must be all of the extra outputs of `plot_configuration()` (excluding the figure which is the first output). The order of these must match the order of the `plot_configuration()` outputs.

```
def update_configuration(x, l, time,  # any variables you need for updating
                         ax, spring_line, square):  # returned items from plot_
↪configuration() in same order

    ax.set_title('{:1.2f} [s]'.format(time))

    xs, ys = spring(0.0, x, l/2, l/2, l/8, n=3)
    spring_line.set_data(xs, ys)

    square.set_xy((x, 0.0))

msd_sys.config_plot_update_func = update_configuration
```

Now that the update function is associated, `animate_configuration()` will create the animation. Here the frames-per-second are set to an explicit value.

```
animation = msd_sys.animate_configuration(fps=30)
```



If using the notebook interactively with `%matplotlib widget` set, the animation above will play. But `animate_configuration()` returns a matplotlib `FuncAnimation` object which has other options that allow the generation of different formats, see https://matplotlib.org/api/_as_gen/matplotlib.animation.FuncAnimation.html for options. One option is to create a Javascript/HTML versions that displays nicely in the notebook with different play options:

```
from IPython.display import HTML

HTML(animation.to_jshtml(fps=30))
```

2.6 Response to sinusoidal forcing

The response to a sinusoidal forcing input, i.e.:

$$m\dot{v} + cv + kx = F_o \sin(\omega t)$$

can be simulated with `sinusoidal_forcing_response()`. This works the same as `free_response` except it requires a forcing amplitude and frequency.

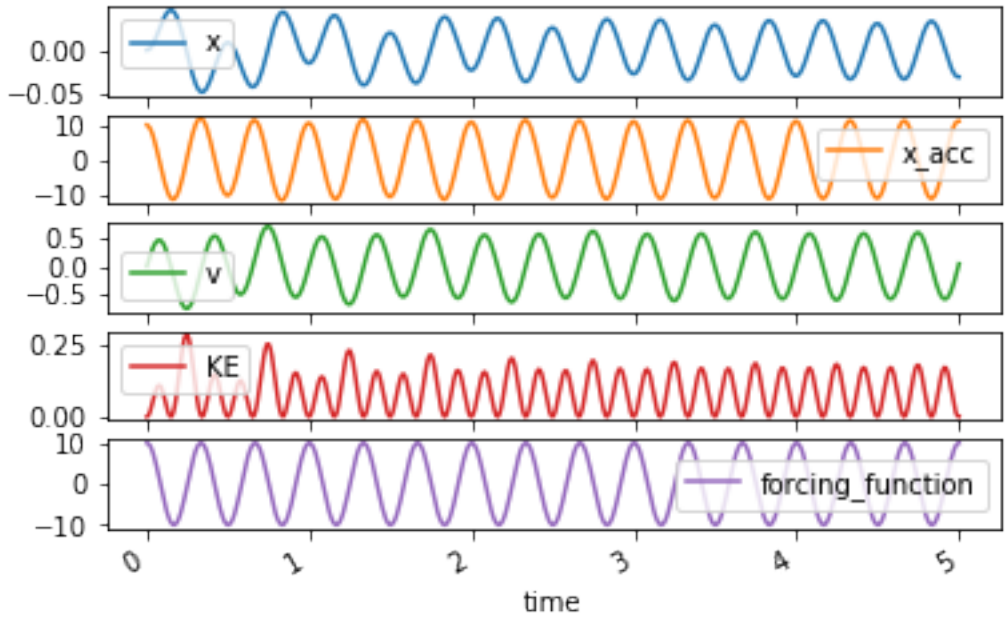
```
msd_sys.coordinates['x'] = 0.0  # m
msd_sys.speeds['v'] = 0.0  # m/s

Fo = 10.0
omega = 2*np.pi*3.0  # rad/s

forced_trajectory = msd_sys.sinusoidal_forcing_response(Fo, omega, 5.0)
```

Note that there is now a `forcing_function` column. This is the applied forcing function.

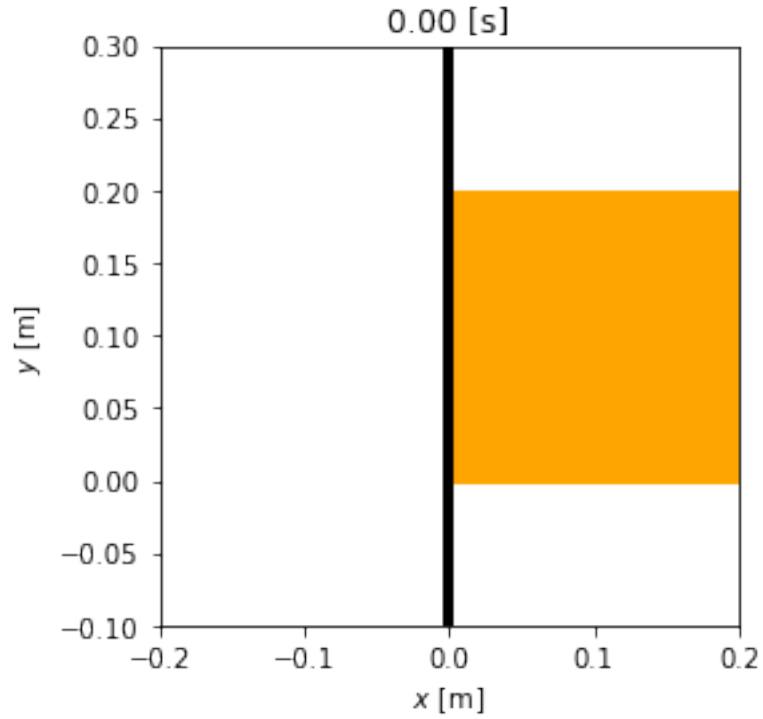
```
forced_trajectory
```

The trajectories can be plotted and animated as above:

```
axes = forced_trajectory.plot(subplots=True)
```




```
fps = 30
animation = msd_sys.animate_configuration(fps=fps)
```

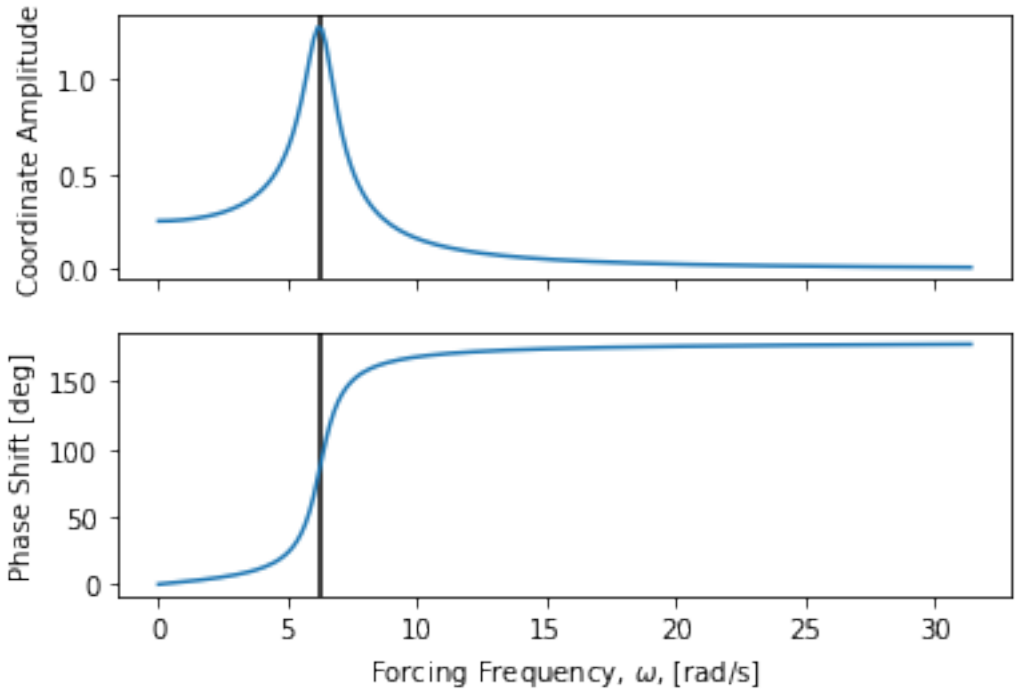



```
HTML(animation.to_jshtml(fps=fps))
```

2.7 Frequency response

The frequency response to sinusoidal forcing at different frequencies can be plotted with `frequency_response_plot()` for a specific forcing amplitude.

```
axes = msd_sys.frequency_response_plot(Fo)
```



2.8 Response to periodic forcing

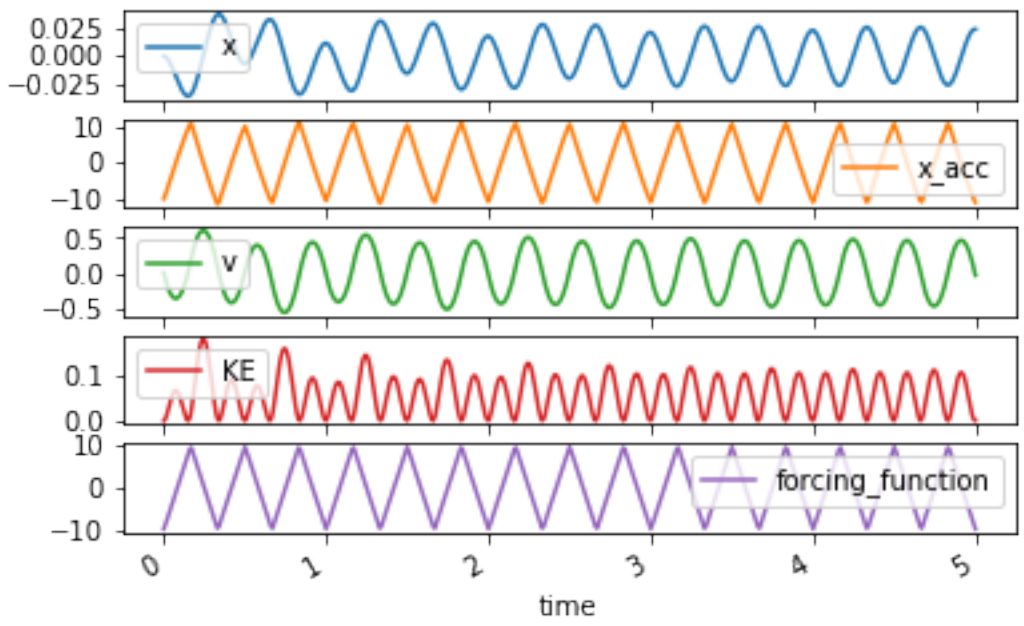
Any periodic forcing function can be applied given the Fourier series coefficients of the approximating function. The following function calculates the Fourier series coefficients for a "sawtooth" shaped periodic input.

```
def sawtooth_fourier_coeffs(A, N):
    """
    A : sawtooth amplitude, Newtons
    T : sawtooth period, seconds
    N : number of Fourier series terms
    """
    n = np.arange(1, N+1)
    an = A*(8*(-1)**n - 8) / 2 / np.pi**2 / n**2
    return 0, an, np.zeros_like(an)

a0, an, bn = sawtooth_fourier_coeffs(Fo, 20)
```

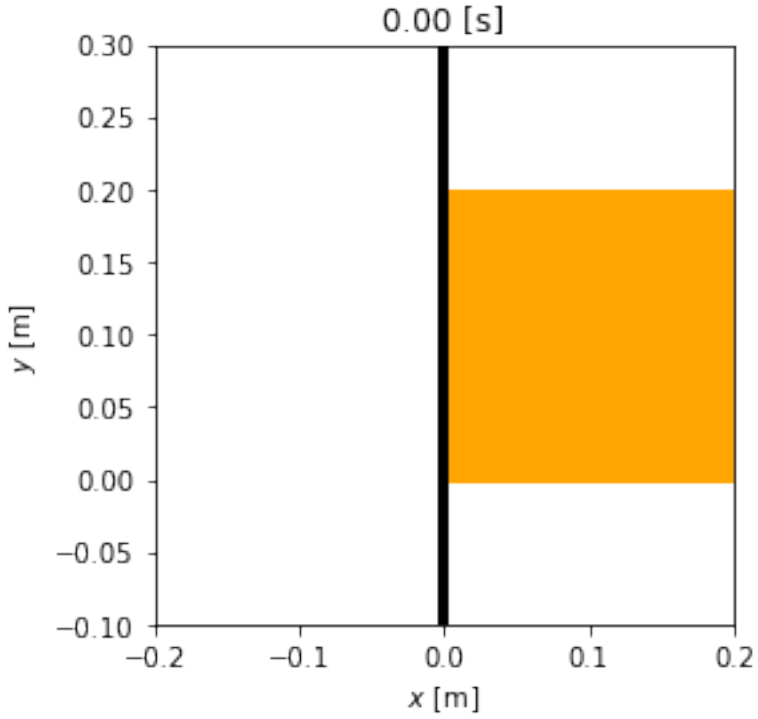
These coefficients can be provided to `periodic_forcing_response()` to simulate the response:

```
wb = 2*np.pi*3.0  # rad/s

trajectory = msd_sys.periodic_forcing_response(a0, an, bn, wb, 5.0)
trajectory
```

```
axes = trajectory.plot(subplots=True)
```




```
fps = 30
animation = msd_sys.animate_configuration(fps=fps)
```




```
HTML(animation.to_jshtml(fps=fps))
```

CHAPTER 3

API

3.1 resonance/system.py

class resonance.system.**System**

This is the abstract base class for any single or multi degree of freedom system. It can be sub-classed to make a custom system or the necessary methods can be added dynamically.

add_measurement(*name*, *func*)

Creates a new measurement entry in the measurements attribute that uses the provided function to compute the measurement given a subset of the constants, coordinates, speeds, other measurements, and time.

Parameters

- **name** (*string*) – This must be a valid Python variable name and it should not clash with any names in the constants, coordinates, or speeds dictionary. This string can be different that the function name.
- **func** (*function*) – This function must only have existing constant, coordinate, speed, or measurement names, and/or the special name `'time'` in the function signature. These can be a subset of the available choices in constants, coordinates, speeds, measurements and any order in the signature is permitted. The function must be able to operate on both inputs that are a collection of floats or a collection of equal length 1D NumPy arrays and floats, i.e. the function must be vectorized. So be sure to use NumPy vectorized functions inside your function, i.e. `numpy.sin()` instead of `math.sin()`. The measurement function you create should return a item, either a scalar or array, that gives the values of the measurement.

Examples

```
>>> import numpy as np
>>> from resonance.linear_systems import SingleDoFLinearSystem
>>> sys = SingleDoFLinearSystem()
>>> sys.constants['m'] = 1.0  # kg
```

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```
>>> sys.constants['c'] = 0.2  # kg*s
>>> sys.constants['k'] = 10.0  # N/m
>>> sys.coordinates['x'] = 1.0  # m
>>> sys.speeds['v'] = 0.25  # m/s
>>> def can_coeffs(m, c, k):
...     return m, c, k
...
>>> sys.canonical_coeffs_func = can_coeffs
>>> def force(x, v, c, k, time):
...    return -k * x - c * v + 5.0 * time
...
>>> # The measurement function you create must be vectorized, such
>>> # that it works with both floats and 1D arrays. For example with
>>> # floats:
>>> force(1.0, 0.5, 0.2, 10.0, 0.1)
-9.6
>>> # And with 1D arrays:
>>> force(np.array([1.0, 1.0]), np.array([0.25, 0.25]), 0.2, 10.0,
...       np.array([0.1, 0.2]))
array([-9.55, -9.05])
>>> sys.add_measurement('f', force)
>>> sys.measurements['f']  # time is 0.0 by default
-10.05
>>> sys.constants['k'] = 20.0  # N/m
>>> sys.measurements['f']
-20.05
>>> # Note that you should use NumPy functions to ensure your
>>> # measurement is vectorized.
>>> sys.constants['force'] = 2.0
>>> def force_mag(force):
...     return np.abs(force)
...
>>> force_mag(-10.05)
10.05
>>> force_mag(np.array([-10.05, -20.05]))
array([10.05, 20.05])
>>> sys.add_measurement('fmag', force_mag)
>>> sys.measurements['fmag']
2.0
```

animate_configuration (*fps=30*, ***kwargs*)

Returns a matplotlib animation function based on the configuration plot and the configuration plot update function.

Parameters

- **fps** (*integer*) – The frames per second that should be displayed in the animation. The latest trajectory will be resampled via linear interpolation to create the correct number of frames. Note that the frame rate will depend on the CPU speed of the computer. You'll likely have to adjust this by trial and error to get something that matches well for your computer if you want the animation to run in real time.
- ****kwargs** – Any extra keyword arguments will be passed to `matplotlib.animation.FuncAnimation()`. The `interval` keyword argument will be ignored.

config_plot_func

The configuration plot function arguments should be any of the system's constants, coordinates, measure-

ments, or `'time'`. No other arguments are valid. The function has to return the matplotlib figure as the first item but can be followed by any number of mutable matplotlib objects that you may want to change during an animation. Refer to the matplotlib documentation for tips on creating figures.

Examples

```
>>> import matplotlib.pyplot as plt
>>> from resonance.linear_systems import SingleDoFLinearSystem
>>> sys = SingleDoFLinearSystem()
>>> sys.constants['radius'] = 5.0
>>> sys.constants['center_y'] = 10.0
>>> sys.coordinates['center_x'] = 0.0
>>> def plot(radius, center_x, center_y, time):
...     fig, ax = plt.subplots(1, 1)
...     circle = plt.Circle((center_x, center_y), radius=radius)
...     ax.add_patch(circle)
...     ax.set_title(time)
...     return fig, circle, ax
...
>>> sys.config_plot_func = plot
>>> sys.plot_configuration()  # doctest: +SKIP
```

config_plot_update_func

The configuration plot update function arguments should be any of the system's constants, coordinates, measurements, or 'time' in any order with the returned values from the `config_plot_func` as the last arguments in the exact order as in the configuration plot return statement. No other arguments are valid. Nothing need be returned from the function. See the matplotlib animation documentation for tips on creating these update functions.

Examples

```
>>> from resonance.linear_systems import SingleDoFLinearSystem
>>> sys = SingleDoFLinearSystem()
>>> sys.constants['radius'] = 5.0
>>> sys.constants['center_y'] = 10.0
>>> sys.coordinates['center_x'] = 0.0
>>> sys.speeds['center_vx'] = 0.0
>>> def calc_coeffs():
...     # dummy placeholder
...     return 1.0, 1.0, 1.0
>>> sys.canonical_coeffs_func = calc_coeffs
>>> def plot(radius, center_x, center_y, time):
...     fig, ax = plt.subplots(1, 1)
...     circle = Circle((center_x, center_y), radius=radius)
...     ax.add_patch(circle)
...     ax.set_title(time)
...     return fig, circle, ax
...
>>> sys.config_plot_function = plot
>>> def update(center_y, center_x, time, circle, ax):
...     # NOTE : that circle and ax have to be the last arguments and be
...     # in the same order as returned from plot()
...     circle.set_xy((center_x, center_y))
...     ax.set_title(time)
```

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```
...
>>> sys.config_plot_update_func = update
>>> sys.free_response(1.0)  #doctest: +SKIP
      center_x  center_x_acc  center_vx
time
0.00       0.0           0.0        0.0
0.01       0.0           0.0        0.0
0.02       0.0           0.0        0.0
0.03       0.0           0.0        0.0
0.04       0.0           0.0        0.0
...        ...           ...        ...
0.96       0.0           0.0        0.0
0.97       0.0           0.0        0.0
0.98       0.0           0.0        0.0
0.99       0.0           0.0        0.0
1.00       0.0           0.0        0.0
```

[101 rows x 3 columns] >>> sys.animate_configuration() #doctest: +SKIP

constants

A dictionary containing the all of the system's constants, i.e. parameters that do not vary with time.

Examples

```
>>> sys = System()
>>> sys.constants
{}
>>> sys.constants['mass'] = 5.0
>>> sys.constants
{'mass': 5.0}
>>> del sys.constants['mass']
>>> sys.constants
{}
>>> sys.constants['mass'] = 5.0
>>> sys.constants['length'] = 10.0
>>> sys.constants
{'mass': 5.0, 'length': 10.0}
```

coordinates

A dictionary containing the system's generalized coordinates, i.e. coordinate parameters that vary with time. These values will be used as initial conditions in simulations.

Examples

```
>>> sys = System()
>>> sys.coordinates['angle'] = 0.0
>>> sys.coordinates
{'angle': 0.0}
```

free_response(*final_time*, *initial_time=0.0*, *sample_rate=100*, *integrator='rungakutta4'*, ***kwargs*)

Returns a data frame with monotonic time values as the index and columns for each coordinate and measurement at the time value for that row. Note that this data frame is stored on the system as the variable `.result` until this method is called again, which will overwrite it.

Parameters

- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **integrator** (*string, optional*) – Either `rungakutta4` or `lsoda`. The `rungakutta4` option is a very simple implementation and the `sample_rate` directly affects the accuracy and quality of the result. The `lsoda` makes use of SciPy's `odeint` function which switches between two integrators for stiff and non-stiff portions of the simulation and is variable step so the sample rate does not affect the quality and accuracy of the result. This has no affect on single degree of freedom linear systems, as their solutions are computed analytically.

Returns **df** – A data frame indexed by time with all of the coordinates and measurements as columns.

Return type pandas.DataFrame

measurements
A dictionary containing the all of the system's measurements, i.e. parameters that are functions of the constants, coordinates, speeds, and other measurements.

plot_configuration()
Returns a matplotlib figure generated by the function assigned to the `config_plot_func` attribute. You may need to call `matplotlib.pyplot.show()` to display the figure.

Returns

- **fig** (*matplotlib.figure.Figure*) – The first returned object is always a figure.
- ***args** (*matplotlib objects*) – Any matplotlib objects can be returned after the figure.

speeds
A dictionary containing the system's generalized speeds, i.e. speed parameters that vary with time. These values will be used as initial conditions in simulations.

Examples

```
>>> sys = System()
>>> sys.speeds['angle_vel'] = 0.0
>>> sys.speeds
{'angle_vel': 0.0}
```

states
An ordered dictionary containing the system's state variables and values. The coordinates are always ordered before the speeds and the individual order of the values depends on the order they were added to coordinates and speeds.

Examples

```
>>> sys = System()
>>> sys.coordinates['angle'] = 0.2
>>> sys.speeds['angle_vel'] = 0.1
>>> sys.states
{'angle': 0.2, 'angle_vel': 0.1}
>>> list(sys.states.keys())
['angle', 'angle_vel']
>>> list(sys.states.values())
[0.2, 0.1]
```

3.2 resonance/linear_systems.py

class resonance.linear_systems.**AutomobileLateralSystem**
Bases: *resonance.linear_systems.MultiDoFLinearSystem*

class resonance.linear_systems.**BallChannelPendulumSystem**
Bases: *resonance.linear_systems.MultiDoFLinearSystem*

class resonance.linear_systems.**BaseExcitationSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents a mass connected to a moving massless base via a spring and damper in parallel. The motion of the mass is subject to viscous damping. The system is described by:

constants

mass, m [kg] The suspended mass.

damping, c [kg / s] The viscous linear damping coefficient which represents any energy dissipation from things like air resistance, friction, etc.

stiffness, k [N / m] The linear elastic stiffness of the spring.

coordinates

position, x [m] The absolute position of the mass.

speeds

velocity, x_dot [m / s] The absolute velocity of the mass.

periodic_base_displacing_response(*twice_avg*, *cos_coeffs*, *sin_coeffs*, *frequency*, *final_time*, *initial_time=0.0*, *sample_rate=100*, *force_col_name='forcing_function'*, *displace_col_name='displacing_function'*)

Returns the trajectory of the system's coordinates, speeds, accelerations, and measurements if a periodic function defined by a Fourier series is applied as displacement of the base in the same direction as the system's coordinate. The displacing function is defined as:

$$y(t) = a0 / 2 + \sum_{n=1}^{N} (an * \cos(n*\omega*t) + bn * \sin(n*\omega*t))$$

Where a0, a1…an, and b1…bn are the Fourier coefficients. If N=∞ then the Fourier series can describe any periodic function with a period (2*π)/ω.

Parameters

- **twice_avg** (*float*) – Twice the average value over one cycle, a0.
- **cos_coeffs** (*float or sequence of floats*) – The N cosine Fourier coefficients: a1, ..., aN.
- **sin_coeffs** (*float or sequence of floats*) – The N sine Fourier coefficients: b1, ..., bN.
- **frequency** (*float*) – The frequency, ω, in radians per second corresponding to one full cycle of the function.
- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **force_col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.
- **displace_col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.

Returns A data frame indexed by time with all of the coordinates, speeds, measurements, and forcing/displacing functions as columns.

Return type pandas.DataFrame

sinusoidal_base_displacing_response(*amplitude*, *frequency*, *final_time*, *initial_time=0.0*, *sample_rate=100*, *force_col_name='forcing_function'*, *displace_col_name='displacing_function'*)

Returns the trajectory of the system's coordinates, speeds, accelerations, and measurements if a sinusoidal displacement function described by:

$y(t) = Y * \sin(\omega * t)$

is specified for the movement of the base in the direction of the system's coordinate.

Parameters

- **amplitude** (*float*) – The amplitude of the displacement function, Y, in meters.
- **frequency** (*float*) – The frequency, ω, in radians per second of the sinusoidal displacement.
- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **force_col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.
- **displace_col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.

Returns A data frame indexed by time with all of the coordinates and measurements as columns.

Return type pandas.DataFrame

class resonance.linear_systems.**BicycleSystem**
Bases: *resonance.linear_systems.MultiDoFLinearSystem*

class resonance.linear_systems.**BookOnCupSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a typical engineering textbook set atop a cylinder (a coffee cup) such that the book can vibrate without slip on the curvature of the cup. It is described by:

constants

thickness, t [meters] the thickness of the book

length, l [meters] the length of the edge of the book which is tangent to the cup's surface

mass, m [kilograms] the mass of the book

radius, r [meters] the outer radius of the cup

coordinates

book_angle, theta [radians] the angle of the book with respect to the gravity vector

speeds

book_angle_vel, theta [radians] the angular rate of the book with respect to the gravity vector

class resonance.linear_systems.**ClockPendulumSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a simple compound pendulum in which a rigid body is attached via a revolute joint to a fixed point. Gravity acts on the pendulum to bring it to an equilibrium state and there is no friction in the joint. It is described by:

constants

pendulum_mass, m [kg] The mass of the compound pendulum.

inertia_about_joint, i [kg m2]** The moment of inertia of the compound pendulum about the revolute joint.

joint_to_mass_center, l [m] The distance from the revolute joint to the mass center of the compound pendulum.

acc_due_to_gravity, g [m/s2]** The acceleration due to gravity.

coordinates

angle, theta [rad] The angle of the pendulum relative to the direction of gravity. When theta is zero the pendulum is hanging down in it's equilibrium state.

speeds

angle_vel, theta_dot [rad / s] The angular velocity of the pendulum about the revolute joint axis.

class resonance.linear_systems.**CompoundPendulumSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a simple compound pendulum in which a rigid body is attached via a revolute joint to a fixed point. Gravity acts on the pendulum to bring it to an equilibrium state and there is no friction in the joint. It is described by:

constants

pendulum_mass, m [kg] The mass of the compound pendulum.

inertia_about_joint, i [kg m2]** The moment of inertia of the compound pendulum about the revolute joint.

joint_to_mass_center, l [m] The distance from the revolute joint to the mass center of the compound pendulum.

acc_due_to_gravity, g [m/s2]** The acceleration due to gravity.

coordinates

angle, theta [rad] The angle of the pendulum relative to the direction of gravity. When theta is zero the pendulum is hanging down in it's equilibrium state.

speeds

angle_vel, theta_dot [rad / s] The angular velocity of the pendulum about the revolute joint axis.

class resonance.linear_systems.**FourStoryBuildingSystem**
Bases: *resonance.linear_systems.MultiDoFLinearSystem*

class resonance.linear_systems.**MassSpringDamperSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a mass connected to a spring and damper (dashpot). The mass moves horizontally without friction and is acted on horizontally by the spring and damper in parallel. The system is described by:

constants

mass, M [kg] The system mass.

damping, C [kg / s] The viscous linear damping coefficient which represents any energy dissipation from things like air resistance, slip, etc.

stiffness, K [N / m] The linear elastic stiffness of the spring.

coordinates
position, x [m]

speeds
velocity, x_dot [m / s]

class resonance.linear_systems.**MultiDoFLinearSystem**
Bases: *resonance.nonlinear_systems.MultiDoFNonLinearSystem*

This is the abstract base class for any multi degree of freedom linear system. It can be sub-classed to make a custom system or the necessary methods can be added dynamically.

canonical_coefficients()
Returns the mass, damping, and stiffness matrices in that order.

canonical_coeffs_func
A function that returns the three linear coefficient matrices of the left hand side of a set of canonical second order ordinary differential equations. This equation looks like the following:

Mv' + Cv + Kx = F(t)

where:

- M: mass matrix
- C: damping matrix
- K: stiffness matrix

- x: the generalized coordinate vector
- v: the generalized speed vector

The coefficients M, C, and K must be defined in terms of the system's constants.

Example

This is an example of a simple double pendulum linearized about its equilibrium.

```
>>> from resonance.linear_systems import MultiDoFLinearSystem
>>> sys = MultiDoFLinearSystem()
>>> sys.constants['g'] = 9.8  # m/s**2
>>> sys.constants['l1'] = 1.0  # m
>>> sys.constants['l2'] = 1.0  # m
>>> sys.constants['m1'] = 0.5  # kg
>>> sys.constants['m2'] = 0.5  # kg
>>> sys.coordinates['theta1'] = 0.3  # rad
>>> sys.coordinates['theta2'] = 0.0  # rad
>>> sys.speeds['omega1'] = 0.0  # rad/s
>>> sys.speeds['omega2'] = 0.0  # rad/s
>>> def coeffs(m1, m2, l1, l2, g):
...     # Represents a linear model of a simple double pendulum
...     M = np.array([[l1 * (m1 + m2), m2 * l2],
...                   [m2 * l2, m2 * l1]])
...     C = np.zeros((2, 2))
...     K = np.array([[-g * (m1 + m2), 0],
...                   [0, -m2 * g]])
...     return M, C, K
>>> sys.canonical_coeffs_func = coeffs
```

forced_response(*final_time*, *initial_time=0.0*, *sample_rate=100*, *integrator='rungakutta4'*, ***kwargs*)

Returns a data frame with monotonic time values as the index and columns for each coordinate and measurement at the time value for that row. Note that this data frame is stored on the system as the variable `result` until this method is called again, which will overwrite it.

Parameters

- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **integrator** (*string, optional*) – Either `rungakutta4` or `lsoda`. The `rungakutta4` option is a very simple implementation and the `sample_rate` directly affects the accuracy and quality of the result. The `lsoda` makes use of SciPy's `odeint` function which switches between two integrators for stiff and non-stiff portions of the simulation and is variable step so the sample rate does not affect the quality and accuracy of the result. This has no affect on single degree of freedom linear systems, as their solutions are computed analytically.

Returns **df** – A data frame indexed by time with all of the coordinates and measurements as columns.

Return type pandas.DataFrame

Notes

You must have defined a `forcing_func` for this to execute. If there is no forcing function this will return the free response.

forcing_func
A function that returns the right hand side forcing vector of the canonical second order linear ordinary differential equations. This equation looks like the following:

Mv' + Cv + Kx = F(t)

where:

- M: mass matrix
- C: damping matrix
- K: stiffness matrix
- x: the generalized coordinate vector
- v: the generalized speed vector

The coefficients M, C, and K must be defined in terms of the system's constants.

Example

This is an example of a simple double pendulum linearized about its equilibrium. The angles, theta1 and theta2, are defined relative to the vertical and when both are zero the pendulum is in its hanging equilibrium. The forcing function applies sinusoidal torquing with respect to theta1 and theta2.

```
>>> from resonance.linear_systems import MultiDoFLinearSystem
>>> sys = MultiDoFLinearSystem()
>>> sys.constants['g'] = 9.8  # m/s**2
>>> sys.constants['l1'] = 1.0  # m
>>> sys.constants['l2'] = 1.0  # m
>>> sys.constants['m1'] = 0.5  # kg
>>> sys.constants['m2'] = 0.5  # kg
>>> sys.coordinates['theta1'] = 0.3  # rad
>>> sys.coordinates['theta2'] = 0.0  # rad
>>> sys.speeds['omega1'] = 0.0  # rad/s
>>> sys.speeds['omega2'] = 0.0  # rad/s
>>> def coeffs(m1, m2, l1, l2, g):
...     # Represents a linear model of a simple double pendulum
...     M = np.array([[l1 * (m1 + m2), m2 * l2],
...                   [m2 * l2, m2 * l1]])
...     C = np.zeros_like(M)
...     K = np.array([[-g * (m1 + m2), 0],
...                   [0, -m2 * g]])
...     return M, C, K
>>> sys.canonical_coeffs_func = coeffs
>>> sys.constants['To'] = 1.0  # Nm
>>> sys.constants['beta'] = 0.01  # rad/s
>>> def forcing(To, beta, time):
...     T1 = To * np.cos(beta * time)
...     T2 = To * np.sin(beta * time)
...     return T1, T2
```

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```
...
>>> sys.forcing_func = forcing
```

class resonance.linear_systems.**SimplePendulumSystem**
Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a simple pendulum in which a point mass is fixed on a massless pendulum arm of some length to a revolute joint. Gravity acts on the pendulum to bring it to an equilibrium state and there is no friction in the joint. It is described by:

constants

pendulum_mass, m [kg] The mass of the compound pendulum.

pendulum_length, l [m] The distance from the revolute joint to the point mass location.

acc_due_to_gravity, g [m/s2]** The acceleration due to gravity.

coordinates

angle, theta [rad] The angle of the pendulum relative to the direction of gravity. When theta is zero the pendulum is hanging down in it's equilibrium state.

speeds

angle_vel, theta_dot [rad / s] The angular velocity of the pendulum about the revolute joint axis.

class resonance.linear_systems.**SimpleQuarterCarSystem**
Bases: *resonance.linear_systems.BaseExcitationSystem*

This system represents a mass connected to a moving massless base via a spring and damper in parallel. The motion of the mass is subject to viscous damping. The system is described by:

constants

mass, m [kg] The suspended mass.

damping, c [kg / s] The viscous linear damping coefficient which represents any energy dissipation from things like air resistance, friction, etc.

stiffness, k [N / m] The linear elastic stiffness of the spring.

coordinates

position, x [m] The absolute position of the mass.

speeds

velocity, x_dot [m / s] The absolute velocity of the mass.

class resonance.linear_systems.**SingleDoFLinearSystem**
Bases: resonance.linear_systems._LinearSystem

This is the abstract base class for any single degree of freedom linear system. It can be sub-classed to make a custom system or the necessary methods can be added dynamically.

frequency_response(*frequencies*, *amplitude*)
Returns the amplitude and phase shift for simple sinusoidal forcing of the system. The first holds the plot of the coordinate's amplitude as a function of forcing frequency and the second holds a plot of the coordinate's phase shift with respect to the forcing function.

Parameters

- **frequencies**(*array_like, shape(n,)*)–

- **amplitude** (*float*) – The value of the forcing amplitude.

Returns

- **amp_curve** (*ndarray, shape(n,)*) – The amplitude values of the coordinate at different frequencies.
- **phase_curve** (*ndarray, shape(n,)*) – The phase shift values in radians of the coordinate relative to the forcing.

frequency_response_plot(*amplitude*, *log=False*, *axes=None*)

Returns an array of two matplotlib axes. The first holds the plot of the coordinate's amplitude as a function of forcing frequency and the second holds a plot of the coordinate's phase shift with respect to the forcing function.

Parameters

- **amplitude** (*float*) – The value of the forcing amplitude.
- **log** (*boolean, optional*) – If True, the amplitude will be plotted on a semi-log Y plot.

period()

Returns the (damped) period of oscillation of the coordinate in seconds.

periodic_forcing_response(*twice_avg*, *cos_coeffs*, *sin_coeffs*, *frequency*, *final_time*, *initial_time=0.0*, *sample_rate=100*, *col_name='forcing_function'*)

Returns the trajectory of the system's coordinates, speeds, accelerations, and measurements if a periodic forcing function defined by a Fourier series is applied as a force or torque in the same direction as the system's coordinate. The forcing function is defined as:

$$F(t) \text{ or } T(t) = a0 / 2 + \sum_{n=1}^{N} (an \star \cos(n \star \omega \star t) + bn \star \sin(n \star \omega \star t))$$

Where a0, a1…an, and b1…bn are the Fourier coefficients. If N=∞ then the Fourier series can describe any periodic function with a period (2*π)/ω.

Parameters

- **twice_avg** (*float*) – Twice the average value over one cycle, a0.
- **cos_coeffs** (*float or sequence of floats*) – The N cosine Fourier coefficients: a1, …, aN.
- **sin_coeffs** (*float or sequence of floats*) – The N sine Fourier coefficients: b1, …, bN.
- **frequency** (*float*) – The frequency, ω, in radians per second corresponding to one full cycle of the function.
- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.

Returns A data frame indexed by time with all of the coordinates and measurements as columns.

Return type pandas.DataFrame

sinusoidal_forcing_response(*amplitude*, *frequency*, *final_time*, *initial_time=0.0*, *sample_rate=100*, *col_name='forcing_function'*)

Returns the trajectory of the system's coordinates, speeds, accelerations, and measurements if a sinusoidal forcing (or torquing) function defined by:

F(t) = Fo * cos(ω * t)

or

T(t) = To * cos(ω * t)

is applied to the moving body in the direction of the system's coordinate.

Parameters

- **amplitude** (*float*) – The amplitude of the forcing/torquing function, Fo or To, in Newtons or Newton-Meters.
- **frequency** (*float*) – The frequency, ω, in radians per second of the sinusoidal forcing.
- **final_time** (*float*) – A value of time in seconds corresponding to the end of the simulation.
- **initial_time** (*float, optional*) – A value of time in seconds corresponding to the start of the simulation.
- **sample_rate** (*integer, optional*) – The sample rate of the simulation in Hertz (samples per second). The time values will be reported at the initial time and final time, i.e. inclusive, along with times space equally based on the sample rate.
- **col_name** (*string, optional*) – A valid Python identifier that will be used as the column name for the forcing function trajectory in the returned data frame.

Returns A data frame indexed by time with all of the coordinates and measurements as columns.

Return type pandas.DataFrame

class resonance.linear_systems.**TorsionalPendulumSystem**

Bases: *resonance.linear_systems.SingleDoFLinearSystem*

This system represents dynamics of a simple torsional pendulum in which the torsionally elastic member's axis is aligned with gravity and the axis of the torsion member passes through the mass center of an object attached to it's lower end. The top of the torsion rod is rigidly attached to the "ceiling". It is described by:

constants

rotational_inertia, I [kg m2]** The moment of inertia of the object attached to the pendulum.

torsional_damping, C [N s / m] The viscous linear damping coefficient which represents any energy dissipation from things like air resistance, slip, etc.

torsional_stiffness, K [N / m] The linear elastic stiffness coefficient of the torsion member, typically a round slender rod.

coordinates

torsional_angle, theta [rad]

speeds

torsional_angle_vel, theta_dot [rad / s]

3.3 resonance/nonlinear_systems.py

class resonance.nonlinear_systems.**BallChannelPendulumSystem**
Bases: *resonance.nonlinear_systems.MultiDoFNonLinearSystem*

class resonance.nonlinear_systems.**ClockPendulumSystem**
Bases: *resonance.nonlinear_systems.SingleDoFNonLinearSystem*

This system represents dynamics of a compound pendulum representing a clock pendulum. It is made up of a thin long cylindrical rod with a thin disc bob on the end. Gravity acts on the pendulum to bring it to an equilibrium state and there is option Coulomb friction in the joint. It is described by:

constants

bob_mass, m_b [kg] The mass of the bob (a thin disc) on the end of the pendulum.

bob_radius, r [m] The radius of the bob (a thin disc) on the end of the pendulum.

rod_mass, m_r [kg] The mass of the then cylindrical rod.

rod_length, l [m] The length of the rod which connects the pivot joint to the center of the bob.

coeff_of_friction, mu [unitless] The Coulomb coefficient of friction between the materials of the pivot joint.

joint_friction_radius, R [m] The radius of the contact disc at the pivot joint. The joint is assumed to be two flat discs pressed together.

joint_clamp_force, F_N [N] The clamping force pressing the two flat discs together at the pivot joint.

acc_due_to_gravity, g [m/s2]** The acceleration due to gravity.

coordinates

angle, theta [rad] The angle of the pendulum relative to the direction of gravity. When theta is zero the pendulum is hanging down in it's equilibrium state.

speeds

angle_vel, theta_dot [rad / s] The angular velocity of the pendulum about the revolute joint axis.

class resonance.nonlinear_systems.**MultiDoFNonLinearSystem**
Bases: *resonance.system.System*

This is the abstract base class for any single degree of freedom nonlinear system. It can be sub-classed to make a custom system or the necessary methods can be added dynamically.

diff_eq_func
A function that returns the time derivatives of the coordinates and speeds, i.e. computes the right hand side of the explicit first order differential equations. This equation looks like the following for linear motion:

```
dx
-- = f(t, q1, ..., qn, u1, ..., un, p1, p2, ..., p0)
dt
```

where:

- x: [q1, . . . , qn, u1, . . . , un], the "state vector"
- t: a time value
- q: the coordinates
- u: the speeds

- p: any number of constants, O is the number of constants

Your function should be able to operate on 1d arrays as inputs, i.e. use numpy math functions in your function, e.g. `numpy.sin` instead of `math.sin`. Besides the constants, coordinates, and speeds, there is a special variable `time` that you can use to give the current value of time inside your function.

Note: The function has to return the derivatives of the states in the order of the `state` attribute.

Warning: Do not use measurements as a function argument. This may cause causality issues and is not yet supported. You are unlikely to get a correct answer if you use a measurement in this function.

Example

```
>>> sys = SingleDoFNonLinearSystem()
>>> sys.constants['gravity'] = 9.8  # m/s**2
>>> sys.constants['length'] = 1.0  # m
>>> sys.constants['mass'] = 0.5  # kg
>>> sys.constants['omega_b'] = 0.1  # rad/s
>>> sys.coordinates['theta'] = 0.3  # rad
>>> sys.speeds['omega'] = 0.0  # rad/s
>>> sys.states  # note the order!
{'theta': 0.3, 'omega': 0.0}
>>> def rhs(theta, omega, gravity, length, mass, omega_b, time):
...     # Represents a linear model of a simple pendulum under
...     # sinusoidal torquing.
...     #  m * l**2 ω' + m * g * l * sin(θ) = sin(ω_b * t)
...     thetad = omega
...     omegad = (np.sin(omega_b * time) -
...               mass*gravity*length*np.sin(theta)) / mass / length**2
...     return thetad, omegad  # in order of sys.states
>>> sys.diff_eq_func = rhs
```

class resonance.nonlinear_systems.**SingleDoFNonLinearSystem**
Bases: *resonance.nonlinear_systems.MultiDoFNonLinearSystem*

3.4 resonance/functions.py

class resonance.functions.**Phasor**(*init*, *frequency=0*, *growth_rate=0*)
Phasor that can be advanced in time with rotation and growth rates.

Parameters

- **init** (*complex*) – Initial phasor in rectangular form (Re + jIm)
- **frequency** (*float, optional*) – Rotation rate in rad/s.
- **growth_rate** (*float, optional*) – Exponential growth rate (decay if < 0).

t
Current time.

Type float

re
Current real component of the phasor.

Type float

im
Current imaginary component of the phasor.

Type float

radius
Current radius of the phasor.

Type float

angle
Current angle of the phasor.

Type float

trace_t
History of time values (since most recent *clear()*).

Type list

trace_re
History of real component values (since most recent *clear()*).

Type list

trace_im
History of imaginary component values (since most recent *clear()*).

Type list

advance(*dt*)
Advance the phasor by a time step dt.

clear()
Clear trajectories.

classmethod from_eig(*eigvec_component*, *eigval*)
Creates a phasor from an eigenvalue/eigenvector component pair.

Parameters

- **eigvec_component** (*complex*) – A single eigenvector component representing the phasor's initial real/imaginary parts.
- **eigval** (*complex*) – The eigenvector, which specifies the phasor's growth rate (real part) and rotational frequency (imaginary part).

class resonance.functions.**PhasorAnimation**(*fig*, *t*, *phasors*, *re_range=(-1, 1)*, *im_range=(-1, 1)*, *repeat=True*, *repeat_delay=0*, *time_stretch=1*, *blit=True*)
Animation for demonstrating rotating phasors.

Two axes are set up. On top, there is an s-plane to show the real and imaginary components of the phasors. The current phasor "vector" is shown with a thick line, the current endpoint of the vector is shown with a circle, thin lines show the projection of the real part of the phasor down to the bottom of the plane, and the time history of the endpoint of the vectors are shown.

On bottom, the phasors' real components are plotted in time. The plot is rotated so that time is positive downward, and the x axes of the s-plane and the time plots are lined up. The current value is shown with a circle, thin lines show the projection from the top of the plot to the current value, and the time history is plotted.

Parameters

- **fig** (*Figure*) – matplotlib Figure object on which to animate.
- **t** (*array*) – Array of time values at which to plot. Even time spacing is assumed.
- **phasors** (*list*) – List of Phasor objects to advance and plot.
- **re_range** (*tuple, optional*) – Limits of the real axis.
- **im_range** (*tuple, optional*) – Limits of the imaginary axis.
- **repeat** (*bool, optional*) – Specifies whether or not to repeat the animation once it finishes.
- **repeat_delay** (*float, optional*) – Amount of time to wait before repeating the animation in milliseconds.
- **time_stretch** (*float, optional*) – Multiplicative factor of the plotting interval. Increasing *time_stretch* effectively makes the animation slower without affecting the time units.
- **blit** (*bool, optional*) – Specifies whether or not to use blitting.

new_frame_seq()
Return a new sequence of frame information.

resonance.functions.**benchmark_par_to_canonical**(*p*)
Returns the canonical matrices of the Whipple bicycle model linearized about the upright constant velocity configuration. It uses the parameter definitions from [Meijaard2007].

Parameters **p** (*dictionary*) – A dictionary of the benchmark bicycle parameters. Make sure your units are correct, best to ue the benchmark paper's units!

Returns

- **M** (*ndarray, shape(2,2)*) – The mass matrix.
- **C1** (*ndarray, shape(2,2)*) – The damping like matrix that is proportional to the speed, v.
- **K0** (*ndarray, shape(2,2)*) – The stiffness matrix proportional to gravity, g.
- **K2** (*ndarray, shape(2,2)*) – The stiffness matrix proportional to the speed squared, v**2.

References

resonance.functions.**centered_rectangle**(*xy*, *width*, *height*, *angle=0.0*)
Returns the arguments for Rectangle given the x and y coordinates of the center of the rectangle.

Parameters

- **xy** (*tuple of floats*) – The x and y coordinates of the center of the rectangle.
- **width** (*float*) – Width of the rectangle. When angle=0.0 this is along the x axis.
- **height** (*float*) – Height of the rectangle. When angle=0.0 this is along the y axis.
- **angle** (*float*) – Angle of rotation about the z axis in degrees.

Returns

- **xy_ll** (*tuple of floats*) – The x and y coordinates of the lower left hand corner of the rectangle.
- **width** (*float*) – Width of the rectangle. When angle=0.0 this is along the x axis.
- **height** (*float*) – Height of the rectangle. When angle=0.0 this is along the y axis.

- **angle** (*float*) – Angle of rotation about the z axis in degrees.

resonance.functions.**estimate_period**(*time*, *signal*)

Computes the period of oscillation based on the given periodic signal.

Parameters

- **time** (*array_like, shape(n,)*) – An array of monotonically increasing time values.
- **signal** (*array_like, shape(n,)*) – An array of values for the periodic signal at each time in `t`.

Returns **period** – An estimate of the period of oscillation.

Return type float

resonance.functions.**spring**(*xA*, *xB*, *yA*, *yB*, *w*, *n=1*, *x=None*, *y=None*)

Returns the x and y coordinates of the points that define a spring diagram between points (xA, yB) and (yA, yB).

Parameters

- **xA** (*float*) – x coordinate of the beginning of the spring.
- **xB** (*float*) – x coordinate of the end of the spring.
- **yA** (*float*) – y coordinate of the beginning of the spring.
- **yB** (*float*) – y coordinate of the end of the spring.
- **w** (*float*) – The width of the spring.
- **n** (*integer, optional*) – Number of coils.
- **x** (*ndarray, shape(2*n + 2), optional*) – Preallocated array for the results.
- **y** (*ndarray, shape(2*n + 2), optional*) – Preallocated array for the results.

Returns

- **x** (*ndarray, shape(2*n + 2)*) – x coordinates of the points that define the ends of each line in the spring.
- **y** (*ndarray, shape(2*n + 2)*) – y coordinates of the points that define the ends of each line in the spring.

Examples

```
>>> import numpy as np
>>> import matplotlib.pyplot as plt
>>> from resonance.functions import spring
>>> plt.axes().set_aspect('equal')
>>> for angle in np.arange(0, 2*np.pi, np.pi/4):
...     plt.plot(*spring(0.0, np.cos(angle),
...                      0.0, np.sin(angle), 0.1, n=4))  # doctest: +SKIP
...
>>> plt.show()
```

CHAPTER 4

Indices and tables

- genindex
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- search

Bibliography

[Meijaard2007] J. P. Meijaard, J. M. Papadopoulos, A. Ruina, and A. L. Schwab, "Linearized dynamics equations for the balance and steer of a bicycle: A benchmark and review," Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, vol. 463, no. 2084, pp. 1955–1982, Aug. 2007.

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